CI²MA

Centro de Investigación en Ingeniería Matemática

Seminario de Análisis Numérico y Modelamiento Matemático de Estudiantes

A new Lagrangian-Eulerian approach for hyperbolic PDEs: theorems, numerics and applications

Modeling, mathematical and numerical analysis challenges for the study of hyperbolic PDEs is in the realm of basic and applied sciences related to fluid mechanics, modeling of vehicular traffic flows, and fluid dynamics in porous media flows, just to name a few specific problems. We design a new class of fully-discrete and semi-discrete Lagrangian-Eulerian schemes to approximate nonlinear multidimensional initial value problems for scalar models and multidimensional systems of hyperbolic conservation laws. The approach is also applied to nonlinear balance laws. The method is based on the concept of multidimensional no-flow curves/surfaces/manifolds. Roughly speaking, one reduces the hyperbolic PDE into a family of ODEs along the forward untangled space-time no-flow Lagrangian trajectories. Due to the no-flow framework, there is no need to compute the eigenvalues (exact or approximate values), and in fact there is no need to construct the Jacobian matrix of the hyperbolic flux functions, and thus giving rise to an effective weak CFL-stability condition in the computing practice. We were able to provide a convergence proof towards the entropy solution to the scalar problem. In the case of multidimensional hyperbolic systems of conservation laws, we show that the Lagrangian-Eulerian scheme also satisfies the weak version of the positivity principle proposed by P. Lax and X.-D. Liu. We also found a connection between some of the results of A. Bressan, in the context of local existence and continuous dependence for discontinuous ODEs and the no-flow curves (now viewed as a forward vector field with locally bounded variation). We present numerical computations for nontrivial (local and nonlocal) hyperbolic problems, as such compressible Euler flows with positivity of the density, the Orszag-Tang problem, which is well-known to satisfy the notable involution-constrained partial differential equation div B = 0, a nonstrictly hyperbolic three-phase flow system in porous media with a resonance point, and the classical 3 by 3 shallow-water system (with and without discontinuous bottom topography). We will also briefly present some numerical results in the context of high-performance parallel computing via a MPI environment.

Dr. Eduardo Cardoso de Abreu

Universidad Estadual de Campinas, Brasil

Académico del Departamento de Matemática Aplicada

Instituto de Matemática, Estatística e Computação Científica

Miércoles 14 de enero

12:00 - 13:00 horas

Auditorio Hermann Alder Weller, CI²MA, UdeC